IIT-JAM 2021

SECTION – A

Multiple Choice Questions (MCQ)

Q1 – Q10 Carry One Mark Each

Q1. The function $e^{\cos x}$ is Taylor expanded about $x=0$. The coefficient of x^2 is

(a) $-\frac{1}{2}$ (b) $-\frac{e}{2}$ (c) $\frac{e}{2}$ (d) Zero

Ans. : (b)

Solution: $f(x)=e^{\cos x}$

Taylor expansion about $x=x_0$ is

$$f(x)=f(x_0)+(x-x_0)f'(x_0)+\frac{(x-x_0)^2}{2!}f''(x_0)+\ldots\infty$$

$\therefore$ Taylor expansion about $x=0$

$$f(x)=f(0)+xf'(0)+x^2\frac{f''(0)}{2!}+--\infty$$

Coefficient of x^2 is $\frac{f''(0)}{2!}$

$$f(x)=e^{\cos x}\Rightarrow f'(x)=-\sin xe^{\cos x}$$

$$f''(x)=-\left[\cos xe^{\cos x}+\sin x\left(-\sin xe^{\cos x}\right)\right]=\left(-\cos x+\sin^2 x\right)e^{\cos x}$$

$$\therefore f''(0)=(-1+0)e^1=-e$$

$$\therefore \text{coefficients of } x^2=\frac{f''(0)}{2!}=\frac{-e}{2}$$

Q2. Let M be a 2×2 matrix. Its trace is 6 and its determinant has value 8. Its eigenvalues are

(a) 2 and 4 (b) 3 and 3 (c) 2 and 6 (d) -2 and -3

Ans. : (a)

Solution: M is a 2×2 matrix

We know trace $=$ sum of eigenvalues

and determinant $=$ product of eigenvalues

Let eigenvalues are λ_1 and λ_2.

$\therefore \lambda_1 + \lambda_2 = 6$ and $\lambda_1 \lambda_2 = 8$

$\lambda_1 = 2$, $\lambda_2 = 4$

Q3. A planet is in a highly eccentric orbit about a star. The distance of its closest approach is 300 times smaller than its farthest distance from the star. If the corresponding speeds are v_c and v_f, then $\frac{v_c}{v_f}$ is

(a) $\frac{1}{300}$ (b) $\frac{1}{\sqrt{300}}$ (c) $\sqrt{300}$ (d) 300

Ans. : (d)

Solution: Using conservation of angular momentum

$$mv_c r_c = mv_f r_f \Rightarrow \frac{v_c}{v_f} = \frac{r_f}{r_c} = \frac{300 r_c}{r_c} = 300$$

Q4. An object of density ρ is floating in a liquid with 75% of its volume submerged. The density of the liquid is

(a) $\frac{4}{3}\rho$ (b) $\frac{3}{2}\rho$ (c) $\frac{8}{5}\rho$ (d) 2ρ

Ans. : (a)

Solution: Weight of object equal to buoyancy force

$$V\rho g = \frac{3}{4} V d g \Rightarrow d = \frac{4}{3}\rho$$

Q5. An experiment with a Michelson interferometer is performed in vacuum using a laser of wavelength $610\,nm$. One of the beams of the interferometer passes through a small glass cavity $1.3\,cm$ long. After the cavity is completely filled with a medium of refractive index n, 472 dark fringes are counted to move past a reference line. Given that the speed of light is $3\times10^8\, m/s$, the value of n is

(a) 1.01 (b) 1.04 (c) 1.06 (d) 1.10

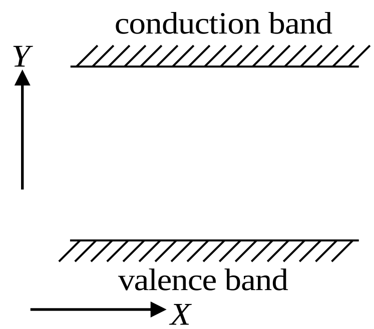
Ans. : (a)

Solution: Change in optical path $\Delta x = m\lambda \Rightarrow 2(n-1)t = m\lambda$

$$\Rightarrow n-1=\frac{n\lambda}{2t}=\frac{472\times 610\times 10^{-9}}{2\times 1\cdot 3\times 10^{-2}}$$

$$\Rightarrow n-1=0\cdot 011\Rightarrow n=1\cdot 011$$

Q6. For a semiconductor material, the conventional flat band energy diagram is shown in the figure. The variables Y, X respectively, are



(a) Energy, Momentum

(b) Energy, Distance

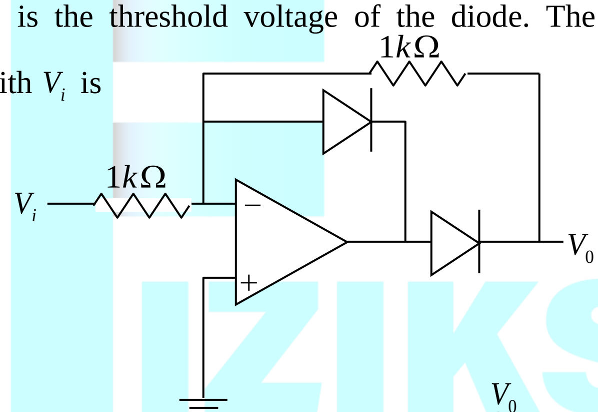
(c) Distance, Energy

(d) Momentum, Energy

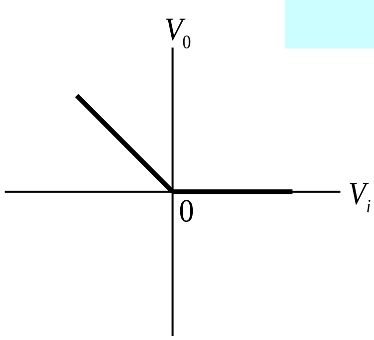
Ans. : (b)

Solution: Along the y-axis energy varies while along x-axis distance is variable.

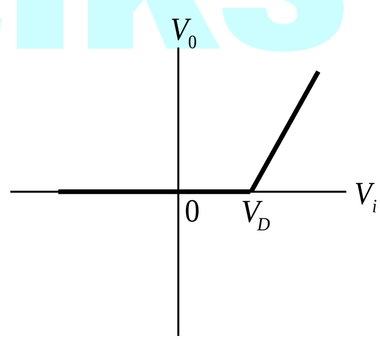
Q7. For the given circuit, V_D is the threshold voltage of the diode. The graph that best deposits variation of V_0 with V_i is



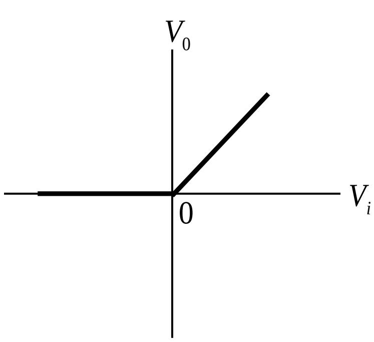
(a)



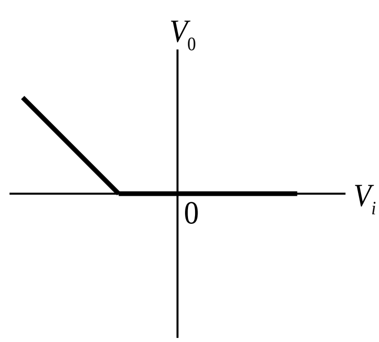
(b)



(c)

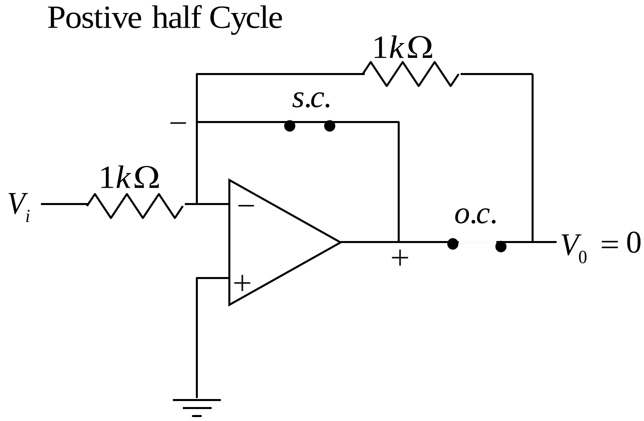


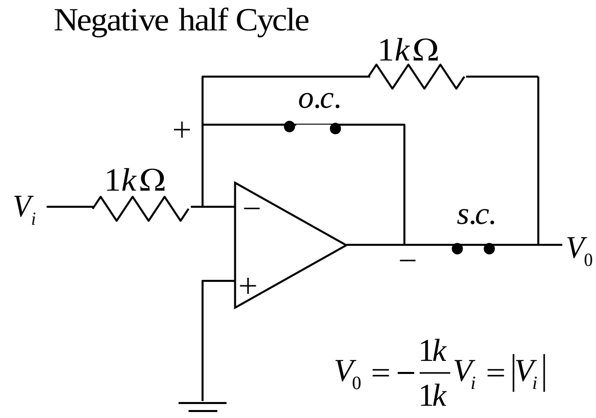
(d)



Ans. : (a)

Solution:





$$V_0 = -\frac{1k}{1k} V_i = |V_i|$$

Q8. Arrange the following telescopes, where D is the telescope diameter and λ is the wavelength, in order of decreasing resolving power:

I. $D = 100\,m,\ \lambda = 21\,cm$ II. $D = 2\,m,\ \lambda = 500\,nm$

III. $D = 1m,\ \lambda = 100\,nm$ IV. $D = 2\,m,\ \lambda = 10\,mm$

(a) III, II, IV, I (b) II, III, I, IV

(c) IV, III, II, I (d) III, II, I, IV

Ans. : (d)

Solution: $d\theta = 1.22\dfrac{\lambda}{D}$

$d\theta_I = 2.56\times10^{-3}$; $d\theta_{II} = 3.05\times10^{-7}$

$d\theta_{III} = 1.22\times10^{-7}$; $d\theta_{IV} = 6.1\times10^{-3}$

Resolving power $= \dfrac{1}{d\theta}$

$RP_{III} > RP_{II} > RP_I > RP_{IV}$

Q9. Metallic lithium has bcc crystal structure. Each unit cell is a cube of side a. The number of atoms per unit volume is

(a) $\dfrac{1}{a^3}$ (b) $\dfrac{2}{\sqrt{2}a^3}$ (c) $\dfrac{2}{a^3}$ (d) $\dfrac{4}{a^3}$

Ans. : (c)

Solution: The effective number of atoms in BCC is $n_{eff} = \frac{1}{8} \times 8 + 1 = 2$

The volume of the bcc unit cell of side a is a^3.

Thus, the number of atoms per unit volume is $= \frac{2}{a^3}$.

Therefore, the correct option is (c).

Q10. The moment of inertia of a solid sphere (radius R and mass M) about the axis which is at a distance of $\frac{R}{2}$ from the centre is

(a) $\frac{3}{20} MR^2$ (b) $\frac{1}{2} MR^2$ (c) $\frac{13}{20} MR^2$ (d) $\frac{9}{10} MR^2$

Ans. : (c)

Solution:

Using parallel axis theorem

$$I_0 = I_{C.M} + Md^2 \; ; \; d = \frac{R}{2} \; , I_{C.M} = \frac{2}{5} MR^2$$

$$I_0 = \frac{2}{5} MR^2 + M \left(\frac{R}{2} \right)^2 = \frac{2}{5} MR^2 + \frac{MR^2}{4} = \left(\frac{8+5}{20} \right) MR^2 = \frac{13}{20} MR^2$$

Q11 – Q30. carry two marks each

Q11. Let (x, y) denote the coordinates in a rectangular Cartesian coordinate system C. Let (x', y') denote the coordinates in another coordinate system C' defined by

$$x' = 2x + 3y$$

$$y' = -3x + 4y$$

The area element in C', is

(a) $\frac{1}{17} dx'dy'$ (b) $12dx'dy'$ (c) $dx'dy'$ (d) $x'dx'dy'$

Ans. : (a)

Solution: $C(x, y) \to C'(x', y')$

$$dxdy = Jdx'dy'$$

where $J = \dfrac{J(x,y)}{J(x',y')} = \begin{vmatrix} \dfrac{\partial x}{\partial x'} & \dfrac{\partial y}{\partial x'} \\ \dfrac{\partial x}{\partial y'} & \dfrac{\partial y}{\partial y'} \end{vmatrix}$

$$J' = \frac{J(x',y')}{J(x,y)} = \begin{vmatrix} \dfrac{\partial x'}{\partial x} & \dfrac{\partial x'}{\partial y} \\ \dfrac{\partial y'}{\partial x} & \dfrac{\partial y'}{\partial y} \end{vmatrix} = \frac{1}{J}$$

$$J' = \begin{vmatrix} 2 & 3 \\ -3 & 4 \end{vmatrix} = 8 - (-9) = 17 \qquad \begin{bmatrix} x' = 2x + 3y \\ y' = -3x + 4y \end{bmatrix}$$

$$\therefore J = \frac{1}{J'} = \frac{1}{17} \qquad \therefore dxdy = \frac{1}{17} dx'dy'$$

Q12. Three events, $E_1(ct = 0, x = 0), E_2(ct = 0, x = L)$ and $E_3(ct = 0, x = -L)$ occur, as observed in an inertial frame S. Frame S' is moving with a speed v along the positive x - direction with respect to S. In S', let t_1', t_2', t_3' be the respective times at which E_1, E_2 and E_3 occurred. Then,

(a) $t_2' < t_1' < t_3'$ (b) $t_1' = t_2' = t_3'$ (c) $t_3' < t_1' < t_2'$ (d) $t_3' < t_2' < t_1'$

Ans. : (a)

Solution: $t_1' = \gamma\left(t_1 - \dfrac{v}{c^2}x_1\right) \Rightarrow t_1' = \gamma t$

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) \Rightarrow t_1' = \gamma\left(t - \frac{v}{c^2}L\right)$$

$$t_3' = \gamma\left(t_3 - \frac{v}{c^2}x_3\right) \Rightarrow t_2' = \gamma\left(t - \frac{v}{c^2}(-L)\right) \Rightarrow t_2' = \gamma\left(t + \frac{v}{c^2}L\right)$$

$$t_2' < t_1' < t_3'$$

Q13. The solution $y(x)$ of the differential equation $y\dfrac{dy}{dx} + 3x = 0, y(1) = 0$, is described by

(a) an ellipse (b) a circle (c) a parabola (d) a straight line

Ans. : (a)

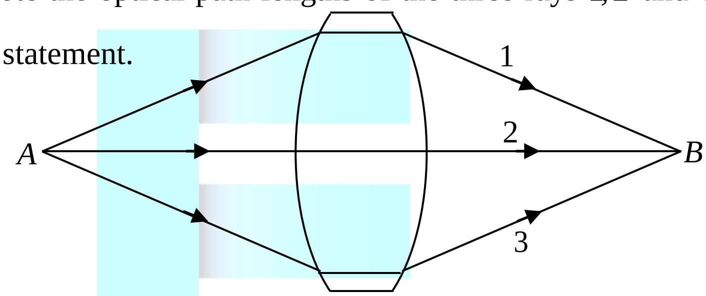
Solution: $y\frac{dy}{dx}+3x=0 \Rightarrow y\frac{dy}{dx}=-3x \Rightarrow \int ydy+\int 3xdx=\int 0 \Rightarrow \frac{y^2}{2}+\frac{3x^2}{2}=c$

Finding the value of c, at $x=1, y=0$; $\Rightarrow \frac{0^2}{2}+\frac{3(1)^2}{2}=c \Rightarrow c=\frac{3}{2}$

$\therefore \frac{y^2}{2}+\frac{3x^2}{2}=\frac{3}{2} \Rightarrow y^2+3x^2=3 \Rightarrow \frac{y^2}{3}+x^2=1 \Rightarrow \frac{x^2}{(1)^2}+\frac{y^2}{\left(\sqrt{3}\right)^2}=1$

Which is equation of an ellipse.

Q14. In the figure below, point A is the object and point B is the image formed by the lens. Let l_1, l_2 and l_3 denote the optical path lengths of the three rays $1, 2$ and 3, respectively. Identify the correct statement.



(a) $l_1=l_2=l_3$ (b) $l_1>l_2<l_3$

(c) $l_1=l_3<l_2$ (d) $l_1=l_3>l_2$

Ans. : (a)

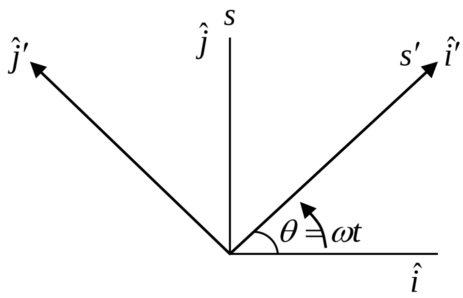
Solution: Fermat's principle: A ray of light in traveling between two points requires either a minimum or a maximum time.

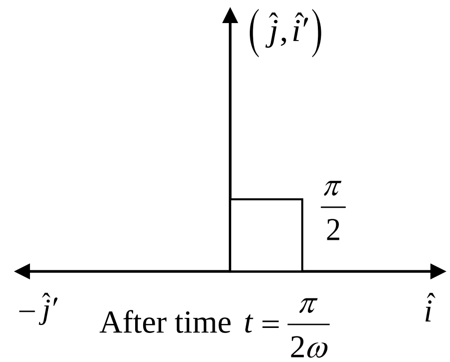
Q15. A particle initially at the origin in an inertial frame S, has a constant velocity $V\hat{i}$. Frame S' is rotating about the z - axis with angular velocity ω (anticlockwise). The coordinate axes of S' coincide with those of S at $t=0$. The velocity of the particle $\left(V_x', V_y'\right)$ in the S' frame, at $t=\frac{\pi}{2\omega}$ is

(a) $\left(-\frac{V\pi}{2}, -V\right)$ (b) $(-V, -V)$ (c) $\left(\frac{V\pi}{2}, -V\right)$ (d) $\left(\frac{3V\pi}{2}, -V\right)$

Ans. : (a)

Solution:





From S frame $V = V\hat{i}$ after time $t = \frac{\pi}{2\omega}$ position vector is $\vec{r} = \vec{V}.t \Rightarrow \vec{r} = \frac{V\pi}{2\omega}\hat{i}$

The angular velocity of $S^{'}$ frame with respect to S frame is $\vec{\omega} = \omega\hat{k}$

Relation between rotation and space frame is

$$\left(\frac{d\vec{r}}{dt}\right)_{Space} = \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\vec{r} \Rightarrow V\hat{i} = \left(\frac{d\vec{r}}{dt}\right)_{rot} + \omega\frac{\pi V}{2\omega}\left(\hat{k}\times\hat{i}\right)$$

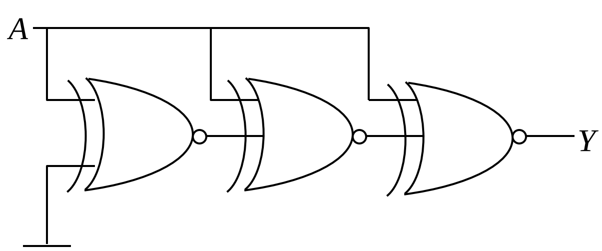
$\left(\frac{d\vec{r}}{dt}\right)_{rot} = V\hat{i} - \frac{\pi V}{2}\hat{j}$ but $\hat{i}$ and $\hat{j}$ defined in S frame.

Due to rotation of $S^{'}$ frame after time $t = \frac{\pi}{2\omega}$; $\hat{i} \equiv -\hat{j}^{'}, \hat{j} \equiv \hat{i}^{'}$

$$\left(\frac{d\vec{r}}{dt}\right)_{rot} = -V\hat{j}^{'} - \frac{\pi V}{2}\hat{i}^{'}$$

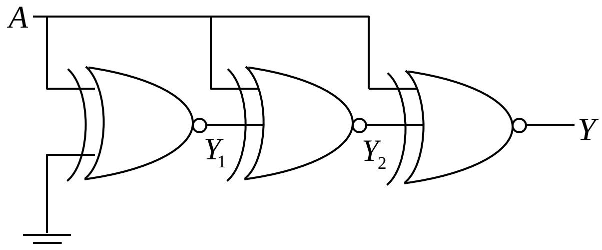
So answer (a) is correct.

Q16. For the given circuit, the output Y is



(a) 0 (b) 1 (c) A (d) $\overline{A}$

Ans. : (d)

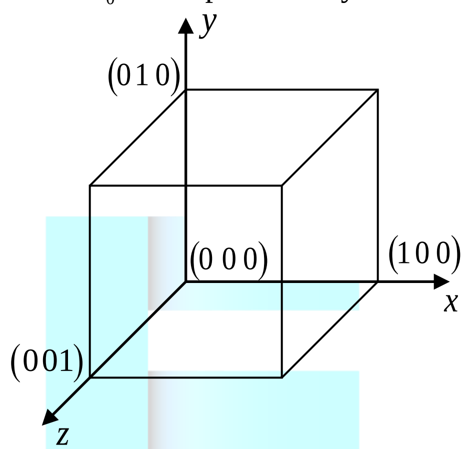


Solution: $Y_1 = \overline{A \oplus 0} = \bar{A}\bar{0} + A0 = \bar{A}$

$Y_2 = \overline{A \oplus \bar{A}} = A\bar{\bar{A}} + \bar{A}A = 0$

Thus $Y = \overline{A \oplus 0} = \bar{A}\bar{0} + A0 = \bar{A}$

Q17. The total charge contained within the cube (see figure), in which the electric field is given by $\vec{E} = K\left(4x^2\hat{i} + 3y\hat{j}\right)$, where ε_0 is the permittivity of free space, is



(a) $7K\varepsilon_0$ (b) $5K\varepsilon_0$ (c) $33K\varepsilon_0$ (d) Zero

Ans. : (a)

Solution: (i) $x = 1, d\vec{a} = dydz\hat{x}, \vec{E}\cdot d\vec{a} = 4Kx^2 dydz = 4Kdydz$ so $\int \vec{E}\cdot d\vec{a} = 4K\int_0^1 dy\int_0^1 dz = 4K$.

(ii) $x = 0, d\vec{a} = -dydz\hat{x}, \vec{E}\cdot d\vec{a} = -4Kx^2 dydz = 0$ so $\int \vec{E}\cdot d\vec{a} = 0$.

(iii) $y = 1, d\vec{a} = dxdz\hat{y}, \vec{E}\cdot d\vec{a} = 3Kydxdz = 3Kdxdz$ so $\int \vec{E}\cdot d\vec{a} = 0$.

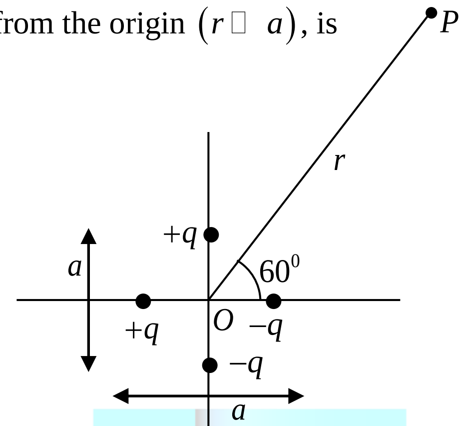
(iv) $y = 0, d\vec{a} = -dxdz\hat{y}, \vec{E}\cdot d\vec{a} = -3Kydxdz = 0$ so $\int \vec{E}\cdot d\vec{a} = 3K\int_0^1 dx\int_0^1 dz = 3K$.

(v) $z = 1, d\vec{a} = dxdy\hat{z}, \vec{E}\cdot d\vec{a} = 0$ so $\int \vec{E}\cdot d\vec{a} = 0$.

(vi) $z = 0, d\vec{a} = -dxdy\hat{z}, \vec{E}\cdot d\vec{a} = 0$ so $\int \vec{E}\cdot d\vec{a} = 0$.

Thus $Q_{enc} = \varepsilon_0 \oint \vec{E}\cdot d\vec{a} = 4K\varepsilon_0 + 0 + 3K\varepsilon_0 + 0 + 0 + 0 = 7K\varepsilon_0$

Q18. Four charges are placed very closed to each other, as shown. The separation between the two charges on the y-axis is a. The separation between the two charges on the x-axis is also a. The leading order (non-vanishing) form of the electrostatic potential, at point P, at a distance r from the origin $(r \gg a)$, is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}\left(\sqrt{3}-1\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2qa}{r^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(\sqrt{5}-1\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(1-\sqrt{3}\right)$

Ans. : (a)

Solution: $Q_{mono} = q+q-q-q=0$

$$\vec{p} = q\times\left(\frac{a}{2}\hat{y}\right)+q\left(-\frac{a}{2}\hat{x}\right)-q\left(-\frac{a}{2}\hat{y}\right)-q\left(\frac{a}{2}\hat{x}\right) = qa\left(-\hat{x}+\hat{y}\right)$$

$$V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\hat{r}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(-\hat{x}.\hat{r}+\hat{y}.\hat{r}\right)$$

$$\Rightarrow V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(-\cos 60^0+\sin 60^0\right) = \frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}\left(\sqrt{3}-1\right)$$

Q19. At $t=0$, N_0 number of a radioactive nuclei A start decaying into B with a decay constant λ_a. The daughter nuclei B decay into nuclei C with a decay constant λ_b. Then, the number of nuclei B at small time t (to the leading order) is

(a) $\lambda_a N_0 t$

(b) $(\lambda_a - \lambda_b) N_0 t$

(c) $(\lambda_a + \lambda_b) N_0 t$

(d) $\lambda_b N_0 t$

Ans. : (a)

Solution: $A \xrightarrow{\lambda_a} B \xrightarrow{\lambda_b} C$

$$N_B = \frac{N_0\lambda_a}{\lambda_b - \lambda_a}\left[e^{-\lambda_a t} - e^{-\lambda_b t}\right] = \frac{N_0\lambda_a}{\lambda_b - \lambda_a}\left[1 - \lambda_a t - 1 + \lambda_b t\right] = \frac{N_0\lambda_a}{\lambda_b - \lambda_a}(\lambda_b - \lambda_a)t$$

$$N_B = N_0\lambda_a t$$

Q20. The electric field of an electromagnetic wave has the form $\vec{E} = E_0\cos(\omega t - kz)\hat{i}$. At $t=0$, a test particle of charge q is at $z=0$, and has velocity $\vec{v} = 0.5c\hat{k}$, where c is the speed of light. The total instantaneous force on the particle is

(a) $\frac{qE_0}{2}\hat{i}$

(b) $\frac{qE_0}{\sqrt{2}}(\hat{i}+\hat{j})$

(c) $\frac{qE_0}{2}(\hat{i}-\hat{k})$

(d) Zero

Ans. : (a)

Solution: $\vec{E} = E_0\cos(\omega t - kz)\hat{i}$ and corresponding $\vec{B} = \frac{E_0}{c}\cos(\omega t - kz)\hat{j}$

Force on charge particle q is $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

$$\vec{F} = qE_0\cos(\omega t - kz)\hat{i} + q\left(\frac{c}{2}\hat{k}\times\frac{E_0}{c}\cos(\omega t - kz)\hat{j}\right)$$

$$\vec{F} = qE_0\cos(\omega t - kz)\hat{i} - \frac{qE_0}{2}\cos(\omega t - kz)\hat{i} = \frac{qE_0}{2}\cos(\omega t - kz)\hat{i}$$

Thus the total instantaneous force on the particle is $\frac{qE_0}{2}\hat{i}$.

Q21. The rms velocity of molecules of oxygen gas is given by v at some temperature T. The molecules of another gas have the same rms velocity at temperature $\frac{T}{16}$. The second gas is

(a) Hydrogen (b) Helium (c) Nitrogen (d) Neon

Ans. : (a)

Solution: $v_{linear} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$, M is molecular mass.

For oxygen, molecule, $M = 2\times16 = 32$

$v_{rms}^2(oxygen) = v_{rms}^2(unknown\ gas) \Rightarrow \frac{3RT}{32} = \frac{3R}{M}\left(\frac{T}{16}\right) \Rightarrow M = 2$ i.e., Hydrogen gas.

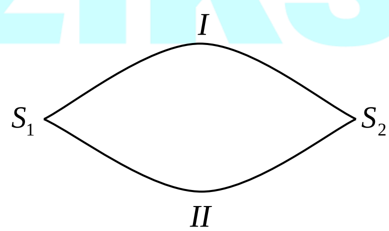
Q22. A system undergoes a thermodynamic transformation from state S_1 to state S_2 via two different paths 1 and 2. The heat absorbed and work done along path 1 are $50\,J$ and $30\,J$, respectively. If the heat absorbed along path 2 is $30\,J$, the work done along path 2 is

(a) Zero (b) $10\,J$ (c) $20\,J$ (d) $30\,J$

Ans. : (b)

Solution: $\Delta Q_I = 50J$, $\Delta W_I = 30J$

$\Delta Q_{II} = 30J$, $\Delta W_{II} = ?$



$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = \Delta Q - \Delta W$

$\Delta U_I = \Delta U_{II} \Rightarrow \Delta Q_I - \Delta W_I = \Delta Q_{II} - \Delta W_{II}$

$\Rightarrow 50 - 30 = 30 - \Delta W_{II} \Rightarrow \Delta W_{II} = 10J$

Q23. The condition for maxima in the interference of two waves

$$Ae^{i\left(\frac{k_0}{2}\left(\sqrt{3}x+y\right)-\omega t\right)} \text{ and } Ae^{i\left(\frac{k_0}{\sqrt{2}}(x+y)-\omega t\right)}$$

is given in terms of the wavelength λ and m, an integer, by

(a) $\left(\sqrt{3}-\sqrt{2}\right)x+\left(1-\sqrt{2}\right)y = 2m\lambda$ (b) $\left(\sqrt{3}+\sqrt{2}\right)x+\left(1-\sqrt{2}\right)y = 2m\lambda$

(c) $\left(\sqrt{3}-\sqrt{2}\right)x-\left(1-\sqrt{2}\right)y = m\lambda$ (d) $\left(\sqrt{3}-\sqrt{2}\right)x+\left(1-\sqrt{2}\right)y = (2m+1)\lambda$

Ans. : (a)

Solution: $\phi_1 = \frac{k_0}{2}\left(\sqrt{3x} + y\right) - \omega t$, $\quad \phi_2 = \frac{k_0}{\sqrt{2}}\left(x + y\right) - \omega t$

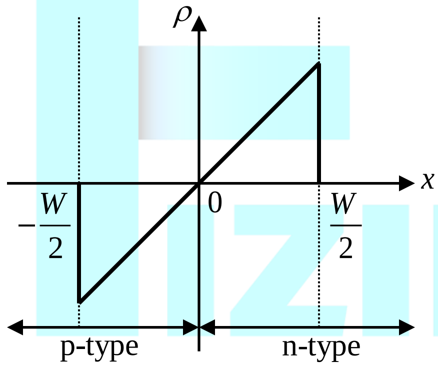
$$\Delta\phi = \phi_1 - \phi_2 = \frac{k_0}{2}\left[\sqrt{3}x + y - \sqrt{2}x - \sqrt{2y}\right] = \frac{k_0}{2}\left[\left(\sqrt{3} - \sqrt{2}\right)x + \left(1 - \sqrt{2}\right)y\right]$$

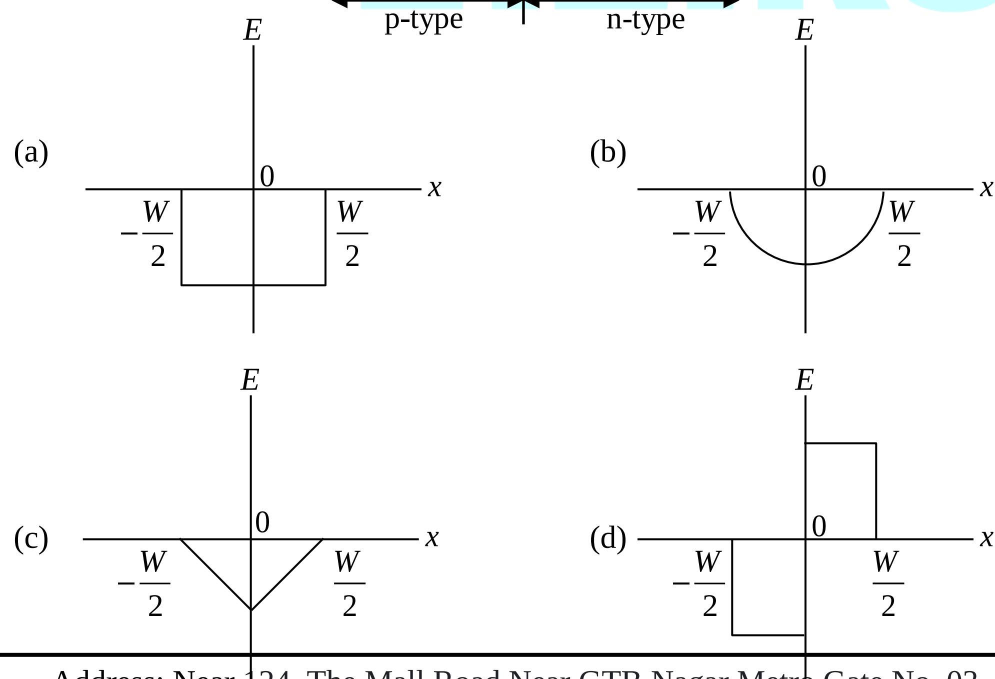
For maxima: $\Delta\phi = 2m\pi$

$$\frac{k_0}{2}\left[\left(\sqrt{3} - \sqrt{2}\right)x + \left(1 - \sqrt{2}\right)y\right] = 2m\pi$$

$$\left(\sqrt{3} - \sqrt{2}\right)x + \left(1 - \sqrt{2}\right)y = \frac{4m\pi}{2\pi/\lambda} = 2m\lambda$$

Q24. A semiconductor pn junction at thermal equilibrium has the space charge density $\rho(x)$ profile as shown in the figure. The figure that best depicts the variation of the electric field E with x is (W denotes the width of the depletion layer)



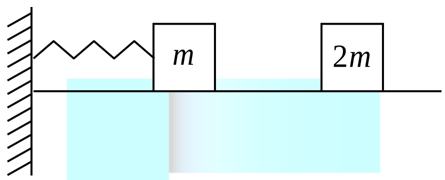


Ans. : (b)

Solution: $\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0} \Rightarrow \frac{dE}{dx} = \frac{\rho}{\varepsilon_0} \propto x \Rightarrow E \propto x^2$

So option (b) is correct.

Q25. A mass m is connected to a massless spring of spring constant k, which is fixed to a wall. Another mass $2m$, having kinetic energy E, collides collinearly with the mass m completely inelastically (see figure). The entire set up is placed on a frictionless floor. The maximum compression of the spring is



(a) $\sqrt{\frac{4E}{3k}}$ (b) $\sqrt{\frac{E}{3k}}$ (c) $\sqrt{\frac{E}{5k}}$ (d) $\sqrt{\frac{E}{7k}}$

Ans. : (a)

Solution: $\frac{1}{2}\left(2mv^2\right) = E \Rightarrow mv^2 = E$

Conservation of momentum: $2m.v = 3m.v_1 \Rightarrow v_1 = \frac{2}{3}v$

Conservation of energy after collision

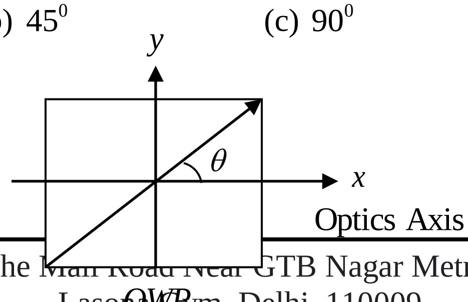
$$\frac{1}{2}3m.v_1^2 = \frac{1}{2}kA^2 \Rightarrow kA^2 = 3m \times \left(\frac{2}{3}v\right)^2 = kA^2 \Rightarrow A = \sqrt{\frac{4mv^2}{3k}} \Rightarrow \sqrt{\frac{4E}{3k}}$$

Q26. A linearly polarized light falls on a quarter wave plate and the emerging light is found to be elliptically polarized. The angle between the fast axis of the quarter wave plate and the plane of polarization of the incident light, can be

(a) 30^0 (b) 45^0 (c) 90^0 (d) 180^0

Ans. : (a)

Solution:



(a) If $\theta = 30^{\circ}$, then emergent light will be EPL.

(b) If $\theta = 45^{0}$, then emergent light will be CPL.

(c) If $\theta = 90^{0}$, than emergent light will be PPL.

(d) If $\theta = 180^{0}$, then emergent light will be PPL.

Q27. The expression for the magnetic field that induces the electric field

$$\vec{E} = K\left(yz\hat{i} + 3z\hat{j} + 4y\hat{k}\right)\cos(\omega t) \text{ is}$$

(a) $-\frac{K}{\omega}\left(\hat{i} + y\hat{j} - z\hat{k}\right)\sin(\omega t)$ (b) $-\frac{K}{\omega}\left(\hat{i} + y\hat{j} + z\hat{k}\right)\sin(\omega t)$

(c) $-\frac{K}{\omega}\left(\hat{i} - y\hat{j} + z\hat{k}\right)\sin(\omega t)$ (d) $-\frac{K}{\omega}\left(-\hat{i} + y\hat{j} + z\hat{k}\right)\sin(\omega t)$

Ans. : (a)

Solution: $\vec{\nabla} \times \vec{E} = K\cos(\omega t)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 3z & 4y \end{vmatrix} = K\cos(\omega t)\left[\hat{i}(4-3) - \hat{j}(0-y) + \hat{k}(0-z)\right]$

$$\Rightarrow \vec{\nabla} \times \vec{E} = K\cos(\omega t)\left[\hat{i} + y\hat{j} - z\hat{k}\right] = -\frac{\partial \vec{B}}{\partial t} \qquad \Rightarrow \vec{B} = -\frac{K}{\omega}sin(\omega t)\left[\hat{i} + y\hat{j} - z\hat{k}\right]$$

Q28. In the Fourier series expansion of two functions $f_1(t) = 4t^2 + 3$ and $f_2(t) = 6t^3 + 7t$ in the interval $-\frac{T}{2}$ to $+\frac{T}{2}$, the Fourier coefficient a_n and b_n (a_n and b_n are coefficients of $\cos(n\omega t)$ and $\sin(n\omega t)$, respectively) satisfy

(a) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n = 0$ for $f_2(t)$

(b) $a_n \neq 0$ and $b_n = 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

(c) $a_n \neq 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

(d) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n \neq 0$ for $f_2(t)$

Ans. : (b)

Solution: Fourier series of a function $b/w \quad x \in \left[\frac{-L}{2}, \frac{L}{2}\right]$

$$g(x) = a_0 + \sum a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

$\therefore$ Fourier series of a function $t \in [-1/2, 1/2]$

$$g(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{T/2} + b_n \sin\frac{n\pi t}{T/2}\right) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\omega t + b_n \sin n\omega t\right)$$

where $a_n = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\cos\frac{2n\pi t}{T}dt$, $\qquad b_n = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\sin\frac{2n\pi t}{T}dt$

For $f_1(t) = 4t^2 + 3$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} \underbrace{f_1(t)}_{even} \underbrace{\cos\frac{2n\pi t}{T}}_{even} \neq 0; \qquad b_n = \frac{2}{T}\int_{-T/2}^{T/2} \underbrace{f_1(t)}_{even} \underbrace{\sin\frac{2n\pi t}{T}}_{odd} = 0$$

For $f_2(t) = 6t^3 + 7t$ (odd function)

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} \underbrace{f_2(t)}_{odd} \underbrace{\cos\frac{2n\pi t}{T}}_{even} dt = 0; \qquad b_n = \frac{2}{T}\int_{-T/2}^{T/2} \underbrace{f_2(t)}_{odd} \underbrace{\sin\frac{2n\pi t}{T}}_{odd} dt \neq 0$$

$\therefore$ For $f_1(t)$: $a_n \neq 0$ and $b_n = 0$ and For $f_2(t)$: $a_n = 0$ and $b_n \neq 0$

Q29. A thin circular disc lying in the xy-plane has a surface mass density σ, given by

$$\sigma(r) = \begin{cases} \sigma_0\left(1 - \frac{r^2}{R^2}\right) & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$$

where r is the distance from its center. Its moment of inertia about the z-axis, passing through its center is

(a) $\frac{\sigma_0 R^4}{4}$ (b) $\frac{\pi\sigma_0 R^4}{6}$ (c) $\sigma_0 R^4$ (d) $2\pi\sigma_0 R^4$

Ans. : (b)

Solution: $I_z = \int_0^{\infty}\int_0^{2\pi} r^2 dm$ where $\sigma(r) = \frac{dm}{rdrd\theta} \Rightarrow dm = \sigma rdrd\theta$

$$I_z = \int_0^\infty \int_0^{2\pi} r^2 \sigma r dr d\theta = 2\pi\sigma_0 \int_0^R r^3 \left(1 - \frac{r^2}{R^2}\right) dr = 2\pi\sigma_0 \left(\frac{R^4}{4} - \frac{R^4}{6}\right) = \frac{\pi\sigma_0 R^4}{6}$$

Q30. The radial component of acceleration in plane polar coordinates is given by

(a) $\frac{d^2r}{dt^2}$

(b) $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$

(c) $\frac{d^2r}{dt^2} + r\left(\frac{d\theta}{dt}\right)^2$

(d) $2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}$

Ans. : (b)

Solution: $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \Rightarrow \frac{dv}{dt} = \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$

$$\Rightarrow a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$$

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q31 – Q40 Carry Two Marks Each

Q31. A gaseous system, enclosed in an adiabatic container, is in equilibrium at pressure P_1 and volume V_1. Work is done on the system in a quasi-static manner due to which the pressure and volume change to P_2 and V_2, respectively, in the final equilibrium state. At every instant, the pressure and volume obey the condition $PV^{\gamma} = C$, where $\gamma = \frac{C_P}{C_V}$ and C is a constant. If the work done is zero, then identify the correct statement(s)

(a) $P_2V_2 = P_1V_1$ (b) $P_2V_2 = \gamma P_1V_1$

(c) $P_2V_2 = (\gamma+1)P_1V_1$ (d) $P_2V_2 = (\gamma-1)P_1V_1$

Ans. : (a)

Solution:

$\Delta Q = 0$ [$\therefore$ the process is adiabatic], $\Delta W = 0$ [given]

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = 0 \Rightarrow \Delta T = 0$

i.e. the process is isothermal too

$\therefore P_1V_1 = P_2V_2$ must satisfy.

Alternate,

Since the process is adiabatic [i.e $\Delta Q = 0$]

And work done, $W = 0$ [given]

We know, for adiabatic process, Work $= \frac{1}{\gamma-1}[P_1V_1 - P_2V_2]$

$$0 = \frac{1}{\gamma-1}(P_1V_1 - P_2V_2) \Rightarrow P_1V_1 = P_2V_2$$

Q32. An isolated ideal gas is kept at pressure P_1 and volume V_1. The gas undergoes free expansion and attains a pressure P_2 and volume V_2. Identify the correct statements(s) $\left(\gamma = \dfrac{C_P}{C_V}\right)$

(a) This is an adiabatic process

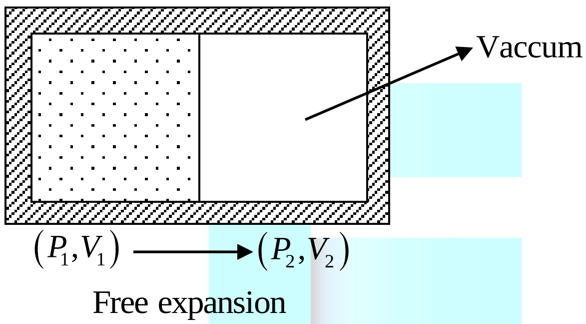
(b) $P_1V_1 = P_2V_2$

(c) $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

(d) This is an isobaric process

Ans. : (a), (b)

Solution:



Free expansion of ideal gas is irreversible adiabatic process. (Since the entropy increase)

Work done in free expansion is zero. (Ideal gas)

$$W = \frac{1}{\gamma - 1}\left(P_2V_2 - P_1V_1\right) = 0 \Rightarrow P_1V_1 = P_2V_2$$

is true for a reversible adiabatic process but since here the process is irreversible, it does not apply here.

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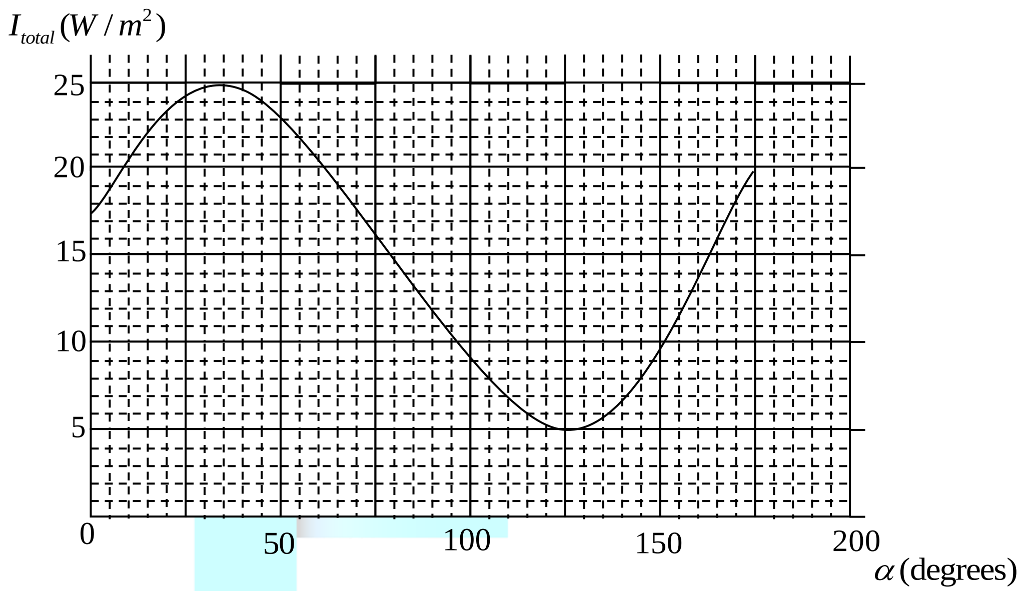
Q33. A beam of light travelling horizontally consists of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization is oriented at an angle θ with respect to the vertical. The figure shows the total intensity I_{total} after the light passes through a polarizer as a function of the angle α, that the axis of the polarizer makes with respect to the vertical. Identify the correct statements(s)



(a) $\theta = 125^0$ (b) $I_p = 5\ W/m^2$

(c) $I_0 = 17.5\ W/m^2$ (d) $I_0 = 10W/m^2; I_p = 20W/m^2$

Ans. : (d)

Solution: $I_{total} = \frac{I_\circ}{2} + I_p \cos^2\theta$, $I_{\min} = \frac{I_\circ}{2} = 5W/m^2 \Rightarrow I_\circ = 10W/m^2$

$I_{\max} = \frac{I_\circ}{2} + I_p = 25 \Rightarrow I_p = 20W/m^2$

Q34. Consider the following differential equation that describes the oscillations of a physical system:

$$\alpha \frac{d^2y}{dt^2} + \beta \frac{dy}{dt} + \gamma y = 0$$

If α and β are held fixed, and γ is increased, then,

(a) The frequency of oscillations increases

(b) The oscillations decay faster

(c) The frequency of oscillations decreases

(d) The oscillations decay slower

Ans. : (a)

Solution: $\frac{d^2y}{dt^2} + \frac{\beta}{\alpha}\frac{dy}{dt} + \frac{\gamma}{\alpha} y = 0$

$2\gamma = \frac{\beta}{\alpha} \Rightarrow \gamma = \frac{\beta}{2\alpha}$, $\omega_0 = \sqrt{\frac{\gamma}{\alpha}}$

(a) $\omega = \sqrt{\omega_0^2 - \gamma^2} = \sqrt{\frac{\gamma}{\alpha} - \frac{\beta^2}{4\alpha^2}} = \frac{1}{2\alpha}\sqrt{4\gamma - \beta^2} \Rightarrow \omega = \frac{1}{2\alpha}\sqrt{4\gamma - \beta^2}$

So, option (a) is correct and option (c) is wrong

(b) $A = A_0 e^{-\gamma t}$

$\gamma = \frac{\beta}{2\alpha} = \text{constant}$

So, option (b) and (d) are is wrong.

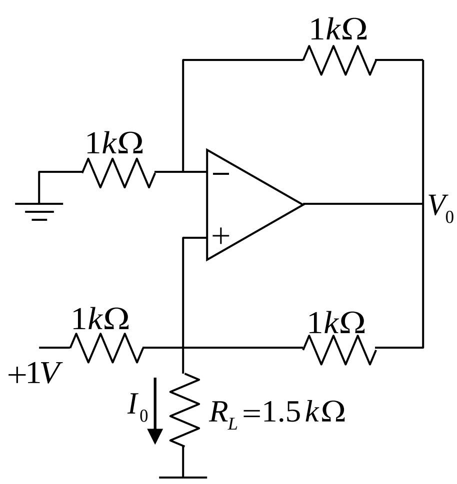
Q35. For the given circuit, identify the correct statement(s)

(a) $I_0 = 1mA$

(b) $V_0 = 3V$

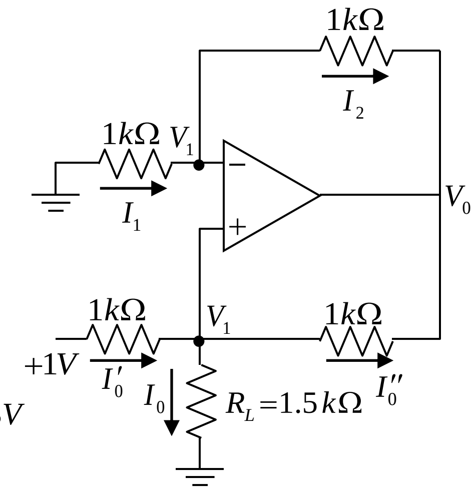
(c) If R_L is doubled, I_0 will change to $0.5mA$

(d) If R_L is doubled, V_0 will change to $6V$



Ans. : (a), (b), (d)

Solution: Apply KCL at inverting terminal



$$I_1 \approx I_2 \Rightarrow \frac{0-V_1}{1k} \approx \frac{V_1 - V_0}{1k} \Rightarrow V_0 = 2V_1$$

Apply KCL at Non-inverting terminal

$$I_0' = I_0 + I_0'' \Rightarrow \frac{1-V_1}{1k} = \frac{V_1}{R_L} + \frac{V_1 - V_0}{1k} \Rightarrow V_1 = R_L \text{ Volts}$$

(a) $I_0 = \frac{V_1}{R_L} = \frac{1.5\,V}{1.5k} = 1mA$ (b) $V_0 = 2V_1 = 2 \times 1.5 = 3V$

(c) $I_0 = \frac{V_1}{R_L} = \frac{3\,V}{3k} = 1mA$ (d) $V_0 = 2V_1 = 6V$

Q36. A Carnot engine operates between two temperatures, $T_L = 100K$ and $T_H = 150K$. Each cycle of the engine lasts for 0.5 seconds during which the power delivered is $500\,J/\text{second}$. Let Q_H be the corresponding heat absorbed by the engine and Q_L be the heat lost. Identify the correct statement(s)

(a) $Q_H = 750\,J$

(b) $\frac{Q_H}{Q_L} \le \frac{2}{3}$

(c) The change in entropy of the engine and the hot bath in a cycle is $5J/K$

(d) The change in entropy of the engine in 0.5 seconds is zero.

Ans. : (a), (c), (d)

Solution: Work done in $1\sec = 500J$

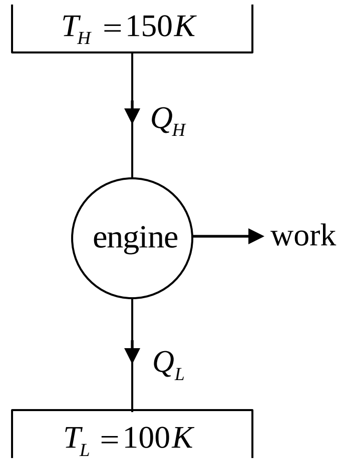
Work done in $0.5\sec = \frac{500}{2} = 250J$ is work done in one cycle $= 250J$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_C}{T_H} = 1 - \frac{100}{150} = \frac{1}{3}$$

$$\eta = \frac{W}{Q_H} \Rightarrow \frac{1}{3} = \frac{250}{Q_H} \Rightarrow Q_H = 750J$$

$$\eta = 1 - \frac{Q_L}{Q_H} = \frac{1}{3} \Rightarrow Q_L = \frac{2}{3} \times 750 = 500J$$

$\Rightarrow \frac{Q_H}{Q_L} = 1.5 > \frac{2}{3}$



Change in entropy of engine in one cycle

$$\Delta S_{engine} = \frac{Q_H}{T_H} + \frac{(-Q_L)}{T_L} = \frac{750}{150} + \frac{(-500)}{100} = 5 - 5 = 0$$

i.e change in entropy of engine in 0.5 seconds is zero.

Change in entropy of engine and hot bath

$$= \Delta S_{engine} + \Delta S_{hot\ bath} = 0 + \frac{(-Q_H)}{T_H} = -5\ J/K$$

Q37. A time independent conservative force $\vec{F}$ has the form, $\vec{F} = 3y\hat{i} + f(x,y)\hat{j}$. Its magnitude at $x = y = 0$ is 8. The allowed form(s) of $f(x,y)$ is (are)

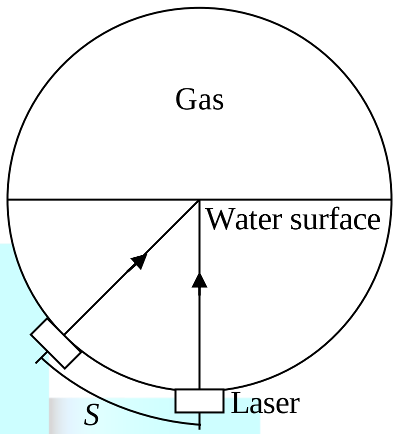
(a) $3x+8$ (b) $2x+8(y-1)^2$ (c) $3x+8e^{-y^2}$ (d) $2x+8\cos y$

Ans. : (a), (c)

Solution: For conservative force $\vec{F}$; $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y & f(x,y) & 0 \end{vmatrix} = 0$

$$\Rightarrow \hat{x}(0-0) + \hat{y}(0-0) + \hat{z}\left(\frac{\partial f}{\partial x} - 3\right) = 0 \Rightarrow \frac{\partial f}{\partial x} = 3$$

Q38. The figure shows the cross-section of hollow cylindrical tank, $2.2\ m$ in diameter which is half filled with water (refractive index of 1.33). The space above the water is filled with a gas of unknown refractive index. A small laser moves along the bottom surface and aims a light beam towards the center (see figure). When the laser moves a distance of $S = 1.09\ m$ or beyond from the lowest point in the water, no light enters the gas. Identify the correct statement(s) (speed of light is $3\times10^{8}\ m/s$).



(a) The refractive index of the gas is 1.05

(b) The time taken for the light beam to travel from the laser to the rim of the tank when $S < 1.09m$ is $8.9 ns$

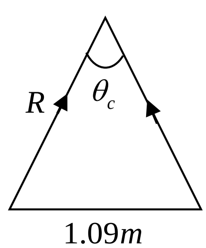
(c) The time taken for the light beam to travel from the laser to the rim of the tank when $S > 1.09m$ is $9.7 ns$

(d) The critical angle for the water-gas interface is 56.77^{0}

Ans. : (b), (c), (d)

Solution: $\theta_c = \frac{S}{R} = \frac{1.09}{1.1} = 0.991\ rad$

$\theta_c = 56.80 \approx 56.77^{0}$



So, option (d) is correct.

(a) Apply Snell's law

$n_\omega \sin\theta_c = n_g \sin 90^{0}$

$n_g = 1.33\times\sin\left(56.77^{0}\right) = 1.11$ or $n_g = 1.33\times\sin\left(56.80\right) = 1.11$

Option (a) is wrong.

(b) If $S < 1.09m$, then time taken by laser light to reach the rim will be

$$t = \frac{R}{v_\omega} + \frac{R}{v_a} = \frac{R}{c}\left(\mu_\omega + \mu_a\right) = \frac{1.1}{3\times 10^8}\left(1.33 + 1.11\right) = \frac{11\times 2.44}{3} ns = 8.94 ns$$

Option (b) is correct.

(c) If $S \geq 1.09m$, then time taken by laser light to reach the rim will be

$$t = \frac{2R}{v_\omega} = \frac{2\times 1.1}{3\times 10^8}\times 1.33 = \frac{22\times 1.33}{3} ns \Rightarrow t = 9.75 ns$$

Option (c) is correct.

Q39. Identify the correct statement(s) regarding nuclei

(a) The uncertainly in the momentum of a proton in a nucleus is roughly 10^5 times the uncertainly in the momentum of the electron in the ground state of Hydrogen atom

(b) The volume of a nucleus grows linearly with the number of nucleons in it

(c) The energy of γ rays due to de-excitation of nucleus can be of the order of MeV

(d) ^{56}Fe is the most stable nucleus

Ans. : (a), (b), (c), (d)

Solution: (a) $\Delta p_p \Delta R_p = \Delta p_e \Delta R_e \approx \frac{\hbar}{2}$

$$\frac{\Delta p_p}{\Delta p_e} = \frac{\Delta R_e}{\Delta R_p} \approx \frac{10^{-10} m}{10^{-15} m} = 10^5$$

(b) $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$

(c) The energy of different energy levels and sublevels is of the order of MeV

(d) Binding energy per nuclei is maximum for ^{56}Fe

Q40. A particle of mass m is in an infinite square well potential of length L. It is in a superposed state of the first two energy eigenstates, as given by $\psi(x) = \frac{1}{\sqrt{3}}|\psi_{n=1}(x) + \sqrt{\frac{2}{3}}\psi_{n=2}(x)$. Identify the correct statement(s). h is Planck's constant.

(a) $\langle p \rangle = 0$ (b) $\Delta p = \frac{\sqrt{3}h}{2L}$ (c) $\langle E \rangle = \frac{3h^2}{8mL^2}$ (d) $\Delta x = 0$

Ans. : (a), (b), (c)

Solution: Incoming momentum and outgoing momentum is same so $\langle p \rangle = 0$

$$\langle E \rangle = \frac{1}{3}E_0 + \frac{2}{3}4E_0 = 3E_0 = \frac{3\pi^2\hbar^2}{2mL^2} \Rightarrow \langle E \rangle = \frac{3h^2}{8mL^2}$$

$$\langle p^2 \rangle = \frac{1}{3}\frac{\pi^2\hbar^2}{L^2} + \frac{2}{3}\frac{4\pi^2\hbar^2}{L^2} = \frac{3\pi^2\hbar^2}{L^2}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \frac{\sqrt{3}h}{2L}$$

$$\Delta x \neq 0$$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q41 – Q50 Carry One Mark Each

Q41. One of the roots of the equation, $z^6 - 3z^4 - 16 = 0$ is given by $z_1 = 2$. The value of the product of the other five roots is ________

Ans. : –8

Solution: $z^6 - 3z^4 - 16 = 0$

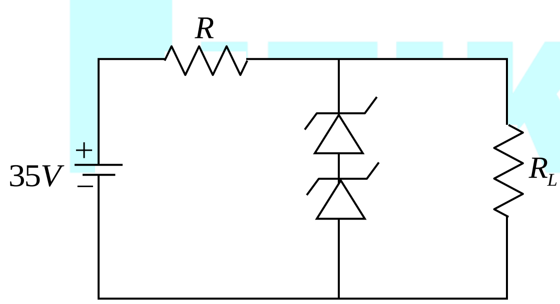
Let the roots be $\alpha, \beta, \gamma, \delta, \varepsilon, \eta$

$\alpha = 2$ (given)

$$\alpha\beta\gamma\delta\varepsilon\eta = \frac{-16}{1} = -16 \Rightarrow \beta\gamma\delta\varepsilon\eta = \frac{-16}{2} - 8$$

i.e product of other five roots $= -8$

Q42. The following Zener diode voltage regulator circuit is used to obtain $20\,V$ regulated output at load resistance R_L from a $35\,V$ dc power supply. Zener diodes are rated at $5W$ and $10\,V$. The value of the resistance R is _________ Ω.



Ans. : 30

Solution: $P_{ZM} = V_Z I_{ZM} \Rightarrow 5W = 10V \times I_{ZM} \Rightarrow I_{ZM} = \frac{1}{2} A$

Current through R is $= \frac{35V - 20V}{R} = \frac{15V}{R}$

Thus $\frac{15V}{R} \geq I_{ZM} = \frac{1}{2} A \Rightarrow R \leq 30\Omega$.

Q43. A small conducting square loop of side l is placed inside a concentric large conducting square loop of side $L(L \gg l)$. The value of mutual inductance of the system is expressed as $\frac{n\mu_0 l^2}{\pi L}$. The value of n is ______ (Round off to two decimal places)

Ans. : 2.828

Solution: Assume smaller loop (loop-1) is at the centre of bigger loop(loop-2). Let current I is flowing in bigger loop.

Magnetic field at the centre of a square loop is $B = 4\times\frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$

$$\Rightarrow B = 4\times\frac{\mu_0 I}{4\pi L/2}\left(\sin 45^0 - \sin\left(-45^0\right)\right) = \frac{4\mu_0 I}{\pi\sqrt{2}L}$$

Flux through loop-1 is $\phi_1 = MI \Rightarrow B\times l^2 = MI \Rightarrow \frac{4\mu_0 I}{\pi\sqrt{2}L}\times l^2 = MI$

$$\Rightarrow M = \frac{4}{\sqrt{2}}\frac{\mu_0 l^2}{\pi L} = 2.828\frac{\mu_0 l^2}{\pi L}$$

Q44. Consider N_1 number of ideal gas particles enclosed in a volume V_1. If the volume is changed to V_2 and the number of particles is reduced by half, the mean free path becomes four times of its initial value. The ratio $\frac{V_1}{V_2}$ is ________ (Round off to one decimal place).

Ans. : 0.5

Solution: $\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{V}{\sqrt{2}N\pi d^2} \Rightarrow \lambda \propto \frac{V}{N} \Rightarrow V \propto \lambda N$

$$\frac{V_1}{V_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \frac{\lambda_1 N_1}{\left(4\lambda_1\right)\left(\frac{N_1}{2}\right)} = 0.5$$

Q45. A particle is moving with a velocity $0.8c\hat{j}$ (c is the speed of light) in an inertial frame S_1. Frame S_2 is moving with a velocity $0.8c\hat{i}$ with respect to S_1. Let E_1 and E_2 be the respective energies of the particle in the two frames. Then, $\frac{E_2}{E_1}$ is ________ (Round off to two decimal places).

Ans. : 1.66

Solution: From S_1 frame the velocity of particle is $0.8c\hat{j}$ so energy is

$$\frac{m_0c^2}{\sqrt{1-\frac{\left|v_{p,S_1}\right|^2}{c^2}}}=\frac{m_0c^2}{\sqrt{1-0.64}}=\frac{10}{6}m_0c^2 \Rightarrow E_1=1.66m_0c^2$$

$u'_x=0$, $u'_y=0.8c$, $v=0.8c$

$$u_x=\frac{u'_x+v}{1+\frac{u'_xv}{c^2}}=0.8c,\ u'_y=\frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{u'_xv}{c^2}}=0.8c\sqrt{1-0.64}=0.8c\times0.6=0.48c$$

$$\left|u_{p,S_2}\right|=\sqrt{(0.8)^2+(0.48)^2}c=\sqrt{0.64+0.23}c=\sqrt{0.87}c=0.93c$$

$$E_2=\frac{m_0c^2}{\sqrt{1-0.87}}=\frac{m_0c^2}{0.36}=2.78m_0c^2 \quad \Rightarrow \frac{E_2}{E_1}=\frac{2.78}{1.67}=1.66$$

Q46. At some temperature T, two metals A and B, have Fermi energies $\in_A$ and $\in_B$, respectively. The free electron density of A is 64 times that of B. The ratio $\frac{\in_A}{\in_B}$ is ____.

Ans. : 16

Solution: The Fermi energy for metal A and B is written as

$$\in_A=\frac{\hbar^2}{2m}\left(3\pi^2n_A\right)^{2/3} \quad \text{and} \quad \in_B=\frac{\hbar^2}{2m}\left(3\pi^2n_B\right)^{2/3}$$

The ratio of the Fermi energy is

$$\frac{\in_A}{\in_B}=\left(\frac{n_A}{n_B}\right)^{2/3}=\left(\frac{64n_B}{n_B}\right)^{2/3}=(64)^{2/3}=16$$

Q47. A crystal has monoclinic structure, with lattice parameters, $a=5.14\ \overset{0}{\mathrm{A}}$, $b=5.20\ \overset{0}{\mathrm{A}}$, $c=5.30\ \overset{o}{A}$ and angle $\beta=99^0$. It undergoes a phase transition to tetragonal structure with lattice parameters, $a = 5.09\,\overset{0}{\mathrm{A}}$ and $c = 5.27\,\overset{0}{\mathrm{A}}$. The fractional change in the volume $\left|\frac{\Delta V}{V}\right|$ of the crystal due to this transition is __________ (Round off to two decimal places).

Ans. : 0.024

Solution: Volume of the monoclinic unit cell is

$$V_{mono} = abc\sin(\beta) = 5.14\times5.20\times5.30\times\sin(99)\times10^{-30}m^3 = 139.91\times10^{-30}m^3$$

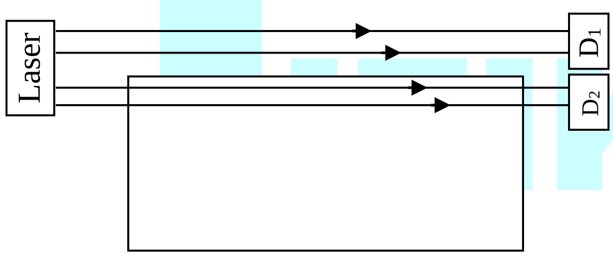
Volume of the tetragonal unit cell is

$$V_{tetr} = a^2c = 5.09\times5.09\times5.27\times10^{-30}m^3 = 136.54\times10^{-30}m^3$$

The fractional change in the volume $\left|\frac{\Delta V}{V}\right|$ is

$$\left|\frac{\Delta V}{V}\right| = \frac{|139.91-136.54|}{139.91} = 0.024$$

Q48. A laser beam shines along a block of transparent material of length $2.5m$. Part of the beam goes to the detector D_1 while the other part travels through the block and then hits the detector D_2. The time delay between the arrivals of the two light beams is inferred to be $6.25ns$. The speed of light $c = 3\times10^8 m/s$. The refractive index of the block is____________ (Round off to two decimal places).



Ans. : 1.73 to 1.77

Solution: Time delay $\Delta t = \frac{\text{Extra path travelled}}{c} \Rightarrow \Delta t = \frac{(\mu-1)x}{c}$

$$(\mu-1) = \frac{\Delta tc}{x} = \frac{6.25\times10^{-9}\times3\times10^8}{2.5} = 0.75 \qquad \Rightarrow \mu = 1.75$$

Q49. An ideal blackbody at temperature T, emits radiation of energy density u. The corresponding value for a material at temperature $\frac{T}{2}$ is $\frac{u}{256}$.

Its emissivity is______________(Round off to three decimal places).

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Ans. : 0.063

Solution: We know $u = \varepsilon\sigma T^4$, where $\varepsilon =$ emissivity

For black body $\varepsilon = 1$.

$\therefore u = \sigma T^4$ (1)

For other material $\frac{u}{256} = \varepsilon\sigma\left(\frac{T}{2}\right)^4 \Rightarrow \frac{u}{256} = \varepsilon\sigma\frac{T^4}{16} \Rightarrow \frac{u}{16} = \varepsilon\sigma T^4$ ....(2)

From (1) and (2)

$\varepsilon = \frac{1}{16} = 0.0625 \approx 0.063$ (Rounded to three decimal places)

Q50. A particle with positive charge $10^{-3}C$ and mass $0.2\,kg$ is thrown upwards from the ground at an angle 45^0 with the horizontal with a speed of $5\,m/s$. The projectile moves through a horizontal electric field of $10\,V/m$, which is in the same direction as the horizontal component of the initial velocity of the particle. The acceleration due to gravity is $10\,m/s^2$. The range is ___________ m. (Round off to three decimal places).

Ans. : 2.51

Solution: $\because y = u_y t - \frac{1}{2}gt^2$, Time of flight $T = \frac{2u_y}{g}$

$\because x = u_x t + \frac{1}{2}\frac{qE}{m}t^2$, Range $R = u_x T + \frac{1}{2}\frac{qE}{m}T^2$

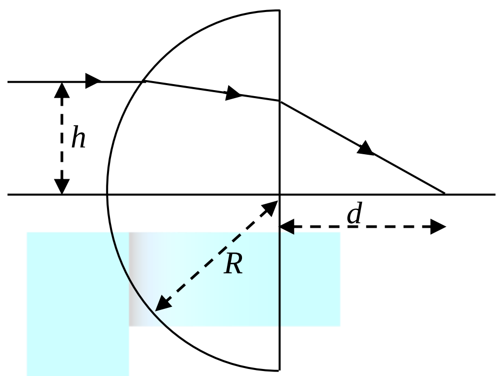
$$R = u_x\frac{2u_y}{g} + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{2u_y}{g}\right)^2$$

where $u_x = \frac{5}{\sqrt{2}}, u_y = \frac{5}{\sqrt{2}}, g = 10, \frac{qE}{m} = \frac{10^{-3}\times 10}{0.2} = \frac{1}{20}$

$$R = \frac{5}{\sqrt{2}}\frac{2\times 5}{\sqrt{2}}\frac{1}{10} + \frac{1}{2}\left(\frac{1}{20}\right)\left(\frac{2\times 5}{\sqrt{2}\times 10}\right)^2 = \frac{25}{10} + \frac{1}{80} = 2.5 + 0.0125 = 2.512$$

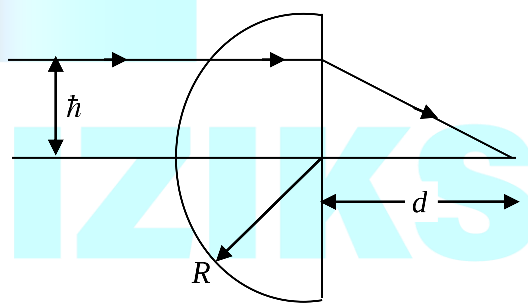
Q51 – Q60 Carry Two Marks Each.

Q51. Consider a hemispherical glass lens (refractive index is 1.5) having radius of curvature $R = 12\,cm$ for the curved surface. An incoming ray, parallel to the optical axis, is incident on the curved surface at a height $h = 1\,cm$ above the optical axis, as shown in the figure. The distance d (from the flat surface of the lens) at which the ray crosses the optical axis is ____________ cm (Round off to two decimal places).



Ans.: 16

Solution: For the curved surface



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

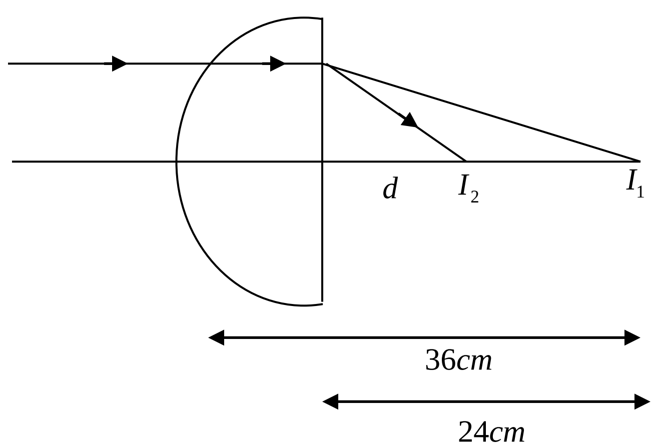
$$\frac{1.5}{v} - \frac{1.0}{\infty} = \frac{1.5 - 1.0}{12}$$

$$\Rightarrow v = 36\,cm$$

For the plane surface

$$\frac{1.0}{d} - \frac{1.5}{24} = \frac{1.0 - 1.5}{\infty}$$

$$\Rightarrow d = 16\,cm$$



Q52. Twenty non-interacting spin $\frac{1}{2}$ particles are trapped in a three-dimensional simple harmonic oscillator potential of frequency ω. The ground state energy of the system, in units of $\hbar\omega$, is __________.

Ans. : 60

Solution: $E=\sum_i 2g_iE_i=(2\times1)\times\frac{3}{2}\hbar\omega+(3\times2)\times\frac{5}{2}\hbar\omega+(2\times6)\times\frac{7}{2}\hbar\omega=(3+15+42)\hbar\omega=60\hbar\omega$

Q53. A thin film of alcohol is spread over a surface. When light from a tunable source is incident normally, the intensity of reflected light at the detector is maximum for $\lambda=640\,nm$ and minimum for $\lambda=512\,nm$. Taking the refractive index of alcohol to be 1.36 for both the given wavelengths, the minimum thickness of the film would be ___________ nm (Round off to two decimal places).

Ans. : 470.58

Solution:

$air\left(\mu_a=1\right)$

$alcohol\left(\mu_{al}=1.36\right)$

$surface\left(\mu_L\right)$

Condition of maxima

$2\mu_L t=n\lambda_1$ (1) $\quad(\lambda=640nm)$

Condition of minima

$2\mu_L t=(2m+1)\lambda_2$ (2) $\quad(\lambda_2=512nm)$

$$\frac{n}{\left(m+\frac{1}{2}\right)}=\frac{\lambda_2}{\lambda_1}=\frac{512}{640}=\frac{4}{5}\Rightarrow\frac{2n}{2m+1}=\frac{2\cdot(2)}{2(2)+1}\qquad\Rightarrow n=m=2$$

From equation (i)

$$2\times1.36\times t=2(640nm)\Rightarrow t=\frac{(2\times640)}{2\times1.36}nm\qquad\Rightarrow t=470.58nm$$

Q54. For the Boolean expression $Y = ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$, the number of combinations for which the output $Y = 1$ is ___________.

Ans. : 4

Solution: $Y = ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

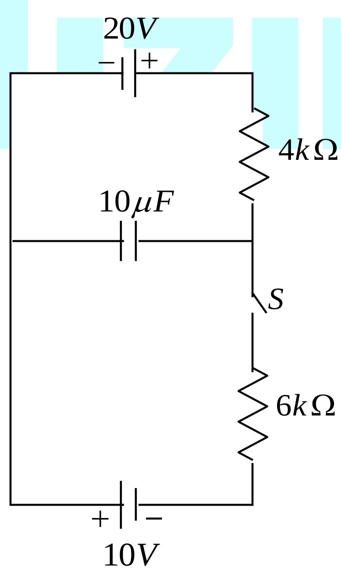
$\Rightarrow Y = \left(AB + \bar{A}\bar{B}\right)C + \left(\bar{A}B + A\bar{B}\right)\bar{C}$

$\Rightarrow Y = DC + \bar{D}\bar{C}$ where $D = AB + \bar{A}\bar{B}$

$\Rightarrow Y = \overline{D \oplus C}$

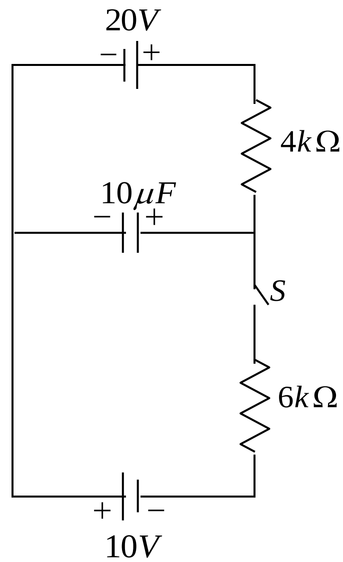
A	B	C	$D = A \oplus B$	$Y = \overline{D \oplus C}$
0	0	0	0	1
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	1
1	1	1	0	0

Q55. An RC circuit is connected to two dc power supplies, as shown in the figure. With switch S open, the capacitor is fully charged. S is then closed at time $t = 0$. The voltage across the capacitor at $t = 2.4$ milliseconds is ___________ V (Round off to one decimal place).



Ans. : 18.824

Solution: Initially capacitor will be charged upto 20 V with polarity shown in figure.



When Switch S is closed, then at $t = 2.4$ milliseconds

$$v_C = V\left(1-e^{-t/RC}\right) = 30\left(1-e^{-\frac{2.4\times10^{-3}}{6\times10^{3}\times10\times10^{3}}}\right)$$

$$\Rightarrow v_C = 30\left(1-e^{-0.04}\right) = 30\left(1-0.9607\right) = 1.176 \text{ Volts}$$

So net voltage across capacitor is

$$=\left(20-1.176\right) \text{ Volts} = 18.824 \text{ Volts}$$

Q56. A current I is uniformly distributed across a long straight nonmagnetic wire ($\mu_r = 1$) of circular cross-section with radius a. Two points P and Q are at distances $\frac{a}{3}$ and $9a$, respectively, from the axis of the wire. The ratio of the magnetic fields at points P and Q is __________.

Ans. : 3

Solution: Magnetic field inside the wire is $B = \frac{\mu_0 Ir}{2\pi a^2} \Rightarrow B_P = \frac{\mu_0 I\left(a/3\right)}{2\pi a^2}$

Magnetic field outside the wire is $B = \frac{\mu_0 I}{2\pi r} \Rightarrow B_Q = \frac{\mu_0 I}{2\pi\left(9a\right)}$

Thus $\frac{B_P}{B_Q} = \frac{\mu_0 I\left(a/3\right)}{2\pi a^2} \times \frac{2\pi\left(9a\right)}{\mu_0 I} = 3$

Q57. A particle A of mass m is moving with a velocity $v\hat{i}$, and collides elastically with a particle B, of mass $2m$, B is initially at rest. After collision, A moves with a velocity $v_A\hat{j}$. If v_B is the final speed of B, then $v_A^2 = kv_B^2$. The value of k is __________.

Ans. : 1

Solution: From conservation of momentum in x-direction

$mv = 2mv_B \cos\theta$ ....(1)

From conservation of momentum in y-direction

$0 = mv_A - 2mv_B \cos\theta \Rightarrow mv_A = 2mv_B \sin\theta$ ....(2)

Squaring and adding both equation

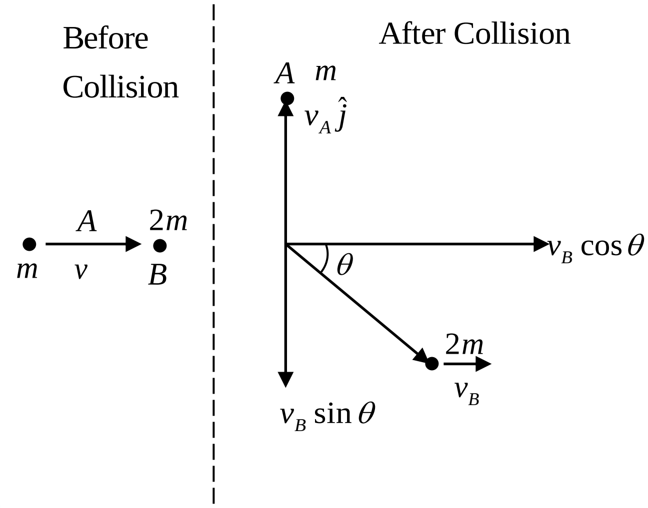
$v^2 + v_A^2 = 4v_B^2$ .... (3)

From conservation of kinetic energy

$\frac{1}{2}mv^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}2mv_B^2 \Rightarrow v^2 = v_A^2 + 2v_B^2$ .... (4)

Solving simultaneously equation (3) and (4)

$v_A^2 = v_B^2 \Rightarrow k = 1$



Q58. In an X- ray diffraction experiment with Cu crystals having lattice parameter $3.61\overset{0}{\text{A}}$, X- rays of wavelength of $0.090\,nm$ are incident on the family of planes $\{1\,1\,0\}$. The highest order present in the diffraction pattern is ___________.

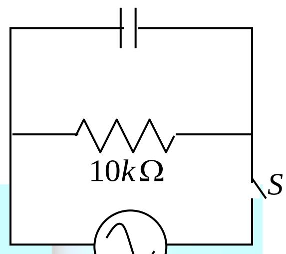
Ans. : 5

Solution: Bragg's law is; $2d\sin(\theta) = n\lambda \quad \Rightarrow \frac{2a}{\sqrt{h^2+k^2+l^2}}\sin(\theta) = n\lambda$

For the highest order $\sin(\theta) = 1$; $n = \frac{2a}{\lambda\sqrt{h^2+k^2+l^2}} = \frac{2\times 3.61\times 10^{-10}}{0.9\times 10^{-10}\times\sqrt{2}} = 5.67$

The maximum order is $n = 5$

Q59. A parallel plate capacitor having plate area of $50\,cm^2$ and separation of $0.1\,mm$ is completely filled with a dielectric (dielectric constant $K=10$). The capacitor is connected to a $10\,k\Omega$ resistance and an alternating voltage $v=10\sin(100\,\pi t)$, as shown in the figure. The switch S is initially open and then closed at $t=0$. The ratio of the displacement current in the capacitor, to the current in the resistance, at time $t=\frac{2}{\pi}$ seconds is ____________ (Round off to three decimal places).



Ans. : 0.038

Solution: Displacement current density $J_d = \varepsilon\frac{\partial E}{\partial t} = \varepsilon_0\varepsilon_r\frac{\partial}{\partial t}\left(\frac{v}{d}\right) = 10\varepsilon_0\frac{\partial}{\partial t}\left(\frac{10\sin(100\,\pi t)}{0.1\times 10^{-3}}\right)$

$J_d = 10^6\varepsilon_0\times 100\,\pi\times\cos(100\,\pi t)$

Displacement current $I_d = J_d\times A = 10^6\varepsilon_0\times 100\,\pi\times\cos(100\,\pi t)\times\left(50\times 10^{-4}\,m^2\right)$ Amp

At time $t=\frac{2}{\pi}$

$I_d = 5\times 10^5\,\pi\varepsilon_0\cos(200)$ Amp $\Rightarrow I_d = 5\times 10^5\times 3.14\times 8.86\times 10^{-12}\times 0.939$ Amp

$\Rightarrow I_d = 130.76\times 10^{-7}$ Amp

Current through resistor R is

$$I_R = \frac{v}{R}\text{ Amp} = \frac{10\sin(100\,\pi t)}{10\times 10^3}\text{ Amp} = \frac{10\sin(200)}{10\times 10^3}\text{ Amp} = 3.42\times 10^{-4}\text{ Amp}$$

Thus $\frac{I_d}{I_R} = \frac{130.76\times 10^{-7}\text{ Amp}}{3.42\times 10^{-4}\text{ Amp}} = 0.038$

Q60. The wavelength of characteristic K_α X - ray photons from Mo (atomic number 42) is ______________ $\overset{0}{\text{A}}$ (Round off to one decimal place).

(Speed of light is $3\times10^8 m/s$; Rydberg constant $R = 1.09\times10^7/m$)

Ans. : 0.73

Solution: $\sqrt{\frac{1}{\lambda_{k\alpha}}} = \sqrt{\frac{3R}{4}}(Z-b)$

$$\lambda_{k\alpha} = \frac{4}{3R(Z-b)^2} = \frac{4}{3\times1.09\times10^7\times(42-1)^2} = 0.73\times10^{-10}m$$

$$\Rightarrow \lambda_{k\alpha} = 0.73A^0$$